Applied Mathematics (104)

Model Answers for Midterm I

Quest-1: Express the following complex Numbers in polar form

(a) 1 (b) i (c) $1/i$ (d) $-1+\sqrt{3}i$ (e) $\frac{1+i}{1-i}$

Sol^n:- If $z=x+iy$ is a complex number then polar form of z is
$z=r(\cos\theta+i\sin\theta)$ where $r=+\sqrt{x^2+y^2}$, $\theta=\tan^{-1}\left(\frac{y}{x}\right)$

Now

(a) since $z=1$ so $r=1$ and $\theta=\tan^{-1}\left(\frac{0}{1}\right)=\tan^{-1}0=0$

Thus $z=1(\cos 0+i\sin 0)$

(b) since $z=i$ so $r=\sqrt{0^2+1^2}=1$ and $\theta=\tan^{-1}\left(\frac{1}{0}\right)=\pi/2$

Thus $z=1\left(\cos\frac{\pi}{2}+i\sin\pi/2\right)$

(c) Since $z=1/i=-i$ so $r=\sqrt{0^2+(-1)^2}=1$ and $\theta=\tan^{-1}\left(\frac{-1}{0}\right)=-\pi/2$

Thus $z=1\left(\cos(-\pi/2)+i\sin(-\pi/2)\right)=\cos\pi/2-i\sin\pi/2$ $\left(\because \cos(-x)=\cos x,\ \sin(-x)=-\sin x\right)$

(d) Since $z=-1+\sqrt{3}i$ so $r=\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{1+3}=2$

and $\theta=\tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)=\tan^{-1}(-\sqrt{3})=2\pi/3$ (since z lies in the IInd quadrant)

Thus $z=2\left(\cos 2\pi/3+i\sin 2\pi/3\right)$

(e) since $z=\frac{1+i}{1-i}=\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{1-1+2i}{2}=i$

Thus $z=1\left(\cos\pi/2+i\sin\pi/2\right)$

Quest-2: Which of the following are true? Justify your answer.

(a) $|z|=|\bar{z}|$ where $z=x+iy$

(b) $AB=0 \Rightarrow A=0$ or $B=0$ where A and B are Matrices.

(c) $AB=BA$ where A and B are Matrices.

Sol^n:- (a) Since $z=x+iy$ Therefore $\bar{z}=x-iy$

again $\because |z|=\sqrt{x^2+y^2}$ and $|\bar{z}|=\sqrt{x^2+(-y)^2}=\sqrt{x^2+y^2}$

So $|z|=|\bar{z}|$

Hence the given statement is true.

(b) $AB = 0 \Rightarrow A = 0$ or $B = 0$

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

clearly $A \neq 0$, and $B \neq 0$

But $AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Therefore the given statement is FALSE.

(c) $AB = BA$

Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

Then $AB = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 3 & 0 \end{bmatrix}$

and ~~Also~~ $BA = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

clearly $AB \neq BA$

Therefore the given statement is FALSE.

Quest-3: If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix}$ and

$(A+B)^2 = A^2 + B^2$, then find x and y.

Sol^n. given that $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix}$

now $A+B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix} = \begin{bmatrix} 1+x & 0 \\ 2+y & -2 \end{bmatrix}$

then $(A+B)^2 = \begin{bmatrix} 1+x & 0 \\ 2+y & -2 \end{bmatrix}\begin{bmatrix} 1+x & 0 \\ 2+y & -2 \end{bmatrix} = \begin{bmatrix} (1+x)^2 & 0 \\ (2+y)(1+x) \\ -4-2y & 4 \end{bmatrix}$

$= \begin{bmatrix} (1+x)^2 & 0 \\ xy-y+2x-2 & 4 \end{bmatrix}$

Now $A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

and $B^2 = \begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix}\begin{bmatrix} x & 1 \\ y & -1 \end{bmatrix} = \begin{bmatrix} x^2+y & x-1 \\ xy-y & y+1 \end{bmatrix}$

Thus $A^2+B^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} x^2+y & x-1 \\ xy-y & y+1 \end{bmatrix}$

$= \begin{bmatrix} x^2+y-1 & x-1 \\ xy-y & y \end{bmatrix}$

Now given that

$$(A+B)^2 = A^2+B^2$$

$$\begin{bmatrix} (x+1)^2 & 0 \\ xy-y+2x-2 & 4 \end{bmatrix} = \begin{bmatrix} x^2+y-1 & x-1 \\ xy-y & y \end{bmatrix}$$

on comparing the coefficients

$$x-1=0$$
$$y=4$$

$\Rightarrow \quad x=1, \quad y=4$

That is the required answer!!

The End!!

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