

: Solution:Q. ① Solution:-

$S^4 + 8S^3 + 18S^2 + 16S + 5 = 0$  Check stability By  
Routh's Array से Routh array method.

$S^4$	1	8	5
$S^3$	8	16	0
$S^2$	16	5	0
$S^1$	13.5	0	0
$S^0$	5	0	0

Result - The system is stable. All the elements of 1<sup>st</sup> row are positive. There is no change sign.

Q. ② Draw polar plot of following system

$$G(s) = \frac{1}{1+s}$$

Solution:-

$$G(s)H(s) = \frac{1}{1+s}$$

$$\Rightarrow G(j\omega)H(j\omega) = \frac{1}{1+j\omega}$$

$$= \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$$

$$|G(j\omega)H(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\omega$$

$$\lim_{\omega \rightarrow 0} |G(j\omega)H(j\omega)| = 1$$

$$\lim_{\omega \rightarrow 0} \angle G(j\omega)H(j\omega) = 0$$

$$\lim_{\omega \rightarrow \infty} |G(j\omega)H(j\omega)| = \lim_{\omega \rightarrow \infty} \frac{1}{\sqrt{1+\omega^2}} = 0$$

$$\lim_{\omega \rightarrow \infty} \angle G(j\omega)H(j\omega) = \lim_{\omega \rightarrow \infty} (-\tan^{-1}\omega) = -90^\circ$$

Polar plot is shown in Fig. 1

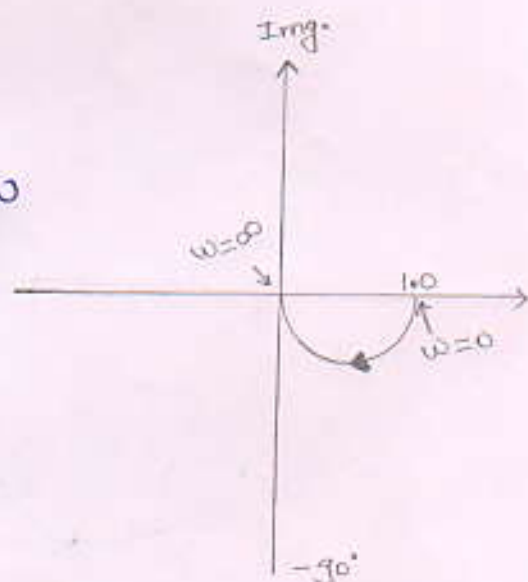


Fig. 1

Q. ③ Open loop transfer - Function of a control system is ②

$$G(s)H(s) = \frac{K}{s(s+6)} \cdot \text{Draw the root locus.}$$

Solution:- There is no open loop zero. The open loop poles are at  $s=0$  and  $s=-6$

1. The root locus starts at  $s=0$  and  $s=-6$  for  $K=0$
2. As there is no open loop zero, so root loci terminates at  $\infty$ .
3. Number of root locus branches  $= N = P = Z_2$  ( $\because P > Z$ )
4. Root locus on real axis line between  $s=0$  to  $s=-6$ .
5. Breakaway point:- It is obtained by solving

$$\frac{dk}{ds} = 0$$

characteristic equation is,

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{s(s+6)} = 0$$

$$\Rightarrow s(s+6) + K = 0$$

$$\Rightarrow s^2 + 6s + K = 0$$

$$\Rightarrow K = -s^2 - 6s$$

$$\Rightarrow \frac{dk}{ds} = -2s - 6 = 0$$

$$\Rightarrow s = -3 \text{ is a breakaway point.}$$

6. angle of asymptotes are given by,

$$\phi = \frac{(2n+1)180^\circ}{p-2} \quad n=0,1$$

$$= \frac{180^\circ}{2}, \frac{3 \times 180^\circ}{2}$$

$$= 90^\circ, 270^\circ (\text{or } -90^\circ)$$

7. Centroid

$$\sigma_A = \frac{\sum \text{real part of poles} - \sum \text{real part of zeros}}{p-2}$$

$$= \frac{(0-6)-0}{2}$$

$$\sigma_A = -3$$

With the data calculated from step 1 to 7, the required ③ root locus is plotted in Fig. 2

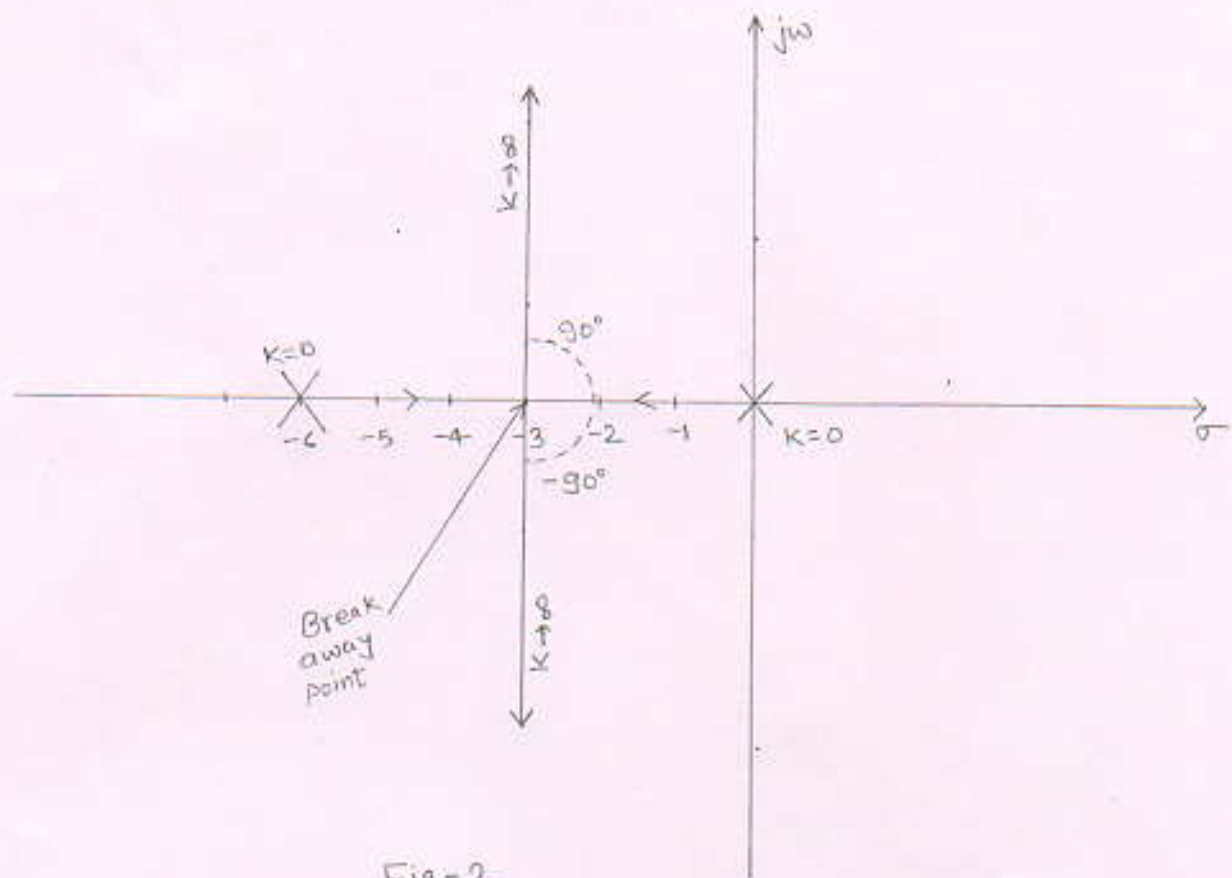


Fig-2