Model answers of questions of Subject Code 104 IInd Class Test session 2017-18

Q.1 How many words can be formed from the letters of the word "KRISHAN"?

Solution :: Number of letters in word "KRISHAN" is 7 and all these are unequal

∴ Number of words can be formed from letters of word "KRISHAN"

$= {}^{n}P_{r}$ where $n=7$ and $r=7$

$= {}^{7}P_{7} = \frac{\lfloor 7}{\lfloor 7-7} = \frac{7\times6\times5\times4\times3\times2\times1}{1}$

Q.2 Out of 10 flowers, in how many ways 4 flowers can be selected for the worship of God?

Solution Number of total flowers $(n) = 10$

Number of flowers selected $(r) = 4$

∴ Number of ways of selection of 4 flowers out of 10 $= {}^{n}C_{r} = {}^{10}C_{4}$

$= \frac{\lfloor 10}{\lfloor 4 \lfloor 10-4} = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$

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Q.3 Find 10th term in the expancion of

$$\left(\frac{a}{2} - \frac{b}{3}\right)^{12}$$

Solution General term in the expansion of

$$(X+A)^n \Rightarrow {}^nc_r \, X^{n-r} A^r = T_{r+1}$$

Here $X = a/2$ $A = -\frac{b}{3}$ $n=12$ $r=9$

$$T_{9+1} = {}^{12}c_9 \left(a/2\right)^{12-9} \left(-b/3\right)^9$$

$$= -\,{}^{12}c_9 \left(a/2\right)^3 \left(b/3\right)^9$$

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Q.4 Show that $e = \frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \cdots$

Solution From exponential series

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots$$

$$e = 2 + \left(\frac{1}{2!} + \frac{1}{3!}\right) + \left(\frac{1}{4!} + \frac{1}{5!}\right) + \cdots$$

$$e = 2 + \left(\frac{3+1}{3!}\right) + \left(\frac{5+1}{5!}\right) + \cdots$$

$$= \frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \cdots$$

OR

Q.4 Prove that

$$\frac{1}{1.2.3} + \frac{1}{3.4.5} + \frac{1}{5.6.7} + \cdots = \log 2 - \frac{1}{2}$$

Solution (i) $\log 2 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots = \frac{1}{1.2} + \frac{1}{3.4}$

(ii) Again $\log 2 = 1 - \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{1}{4} - \frac{1}{5}\right) \cdots = 1 - \frac{1}{2.3} - \frac{1}{4.5}$

$$(i) + (ii) \Rightarrow 2\log 2 = 1 + \left(\frac{1}{1.2} - \frac{1}{2.3}\right) + \left(\frac{1}{3.4} - \frac{1}{4.5}\right) + \cdots$$

$$2\log 2 - 1 = \frac{2}{1.2.3} + \frac{2}{3.4.5} + \cdots$$

$$\log 2 - \frac{1}{2} = \frac{1}{1.2.3} + \frac{1}{3.4.5} + \cdots$$

Hence Proved

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Q(5) Write following complex number in polar form $-1+\sqrt{-3}$

Solution Given complex number $= -1+\sqrt{-3}$
$= -1+\sqrt{3}\,i$

let $r\cos\theta = -1$ ~~rcs~~ $r\sin\theta = \sqrt{3}$

then $r^2 = (-1)^2 + (\sqrt{3})^2$

or $r = 2$ —— (i)

and $\tan\theta = \dfrac{\sqrt{3}}{-1}$

or $\theta = 2\pi/3$ —— (ii)

from (i) and (ii)

$$-1+i\sqrt{3} = 2\left(\cos 2\pi/3 + i\sin 2\pi/3\right)$$

OR

Q.(5) Write following complex number in polar form $\dfrac{1+i}{\sqrt{2}}$

Solution let $r\cos\theta = \dfrac{1}{\sqrt{2}}$ $r\sin\theta = \dfrac{1}{\sqrt{2}}$

then $r^2 = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

or $r = 1$

and $\tan\theta = \dfrac{1/\sqrt{2}}{1/\sqrt{2}}$

or $\theta = \pi/4$

from (i) and (ii)

$$\frac{1+i}{\sqrt{2}} = \cos\pi/4 + i\sin\pi/4$$

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Q.⑥ Find the length of intercepts on the line $x \sin\alpha + y\cos\alpha = \sin 2\alpha$ by x and y axis.

Solution We will change the line equation in intercept form of line equation.

$$\frac{x\sin\alpha}{\sin 2\alpha} + \frac{y\cos\alpha}{\sin 2\alpha} = 1$$

$$\frac{x}{2\cos\alpha} + \frac{y}{2\sin\alpha} = 1$$

by comparing with standard form

$$\frac{x}{a} + \frac{y}{b} = 1$$

we get $a = 2\cos\alpha$ and $b = 2\sin\alpha$

OR

Q.⑥ Find the equation of line which is perpendicular bisector of the line joining the points $(2,1)$ and $(4,3)$

Solution Coordinates of middle point of line joining points $(2,1)$ and $(4,3)$

$$= \left(\frac{2+4}{2}, \frac{1+3}{2}\right) \Rightarrow (3,2)$$

and slope of this line $= \frac{3-1}{4-2} = 1$

$\therefore$ slope of desirable line $= -1$

Now equation of line passing through $(3,2)$ and of slope -1

$$y - 2 = -1(x-3)$$

$$x + y - 5 = 0 \quad \text{Ans.}$$

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