CE-202 , IInd Test, 2017-18

Q. 1 A horizontal venturimeter with inlet diameter 30 cm and throat diameter 15 cm is used to measure the flow of an oil with a specific gravity 0.8. The discharge of the oil through venturimeter is 50 ltr/s. Find the readings of the oil-mercury differential manometer. Take $C_d = 0.98$

Soln. Given $d_1 = 30$ cm $= 0.30$ m, $d_2 = 15$ cm $= 0.15$ m

$Q = 50$ ltr/s $= 0.05\ m^3/s$

$S = 0.8$, $S_m = 13.6$, $C_d = 0.98$

$$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.30)^2 = 0.0707\ m^2$$

$$a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.15)^2 = 0.0177\ m^2$$

We know $$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$0.05 = 0.98 \times \frac{0.0707 \times 0.0177}{\sqrt{(0.0707)^2 - (0.0177)^2}} \times \sqrt{2 \times 9.81 \times$$

$$\sqrt{h} = 0.63$$

$$h = (0.63)^2 = 0.3969\ m$$

As $h = x\left(\frac{S_m}{S} - 1\right)$; $0.3969 = x\left(\frac{13.6}{0.8} - 1\right)$

$$x = 0.0248\ m = 24.8\ mm$$

∴ Reading of oil-mercury diff manometer = 24.8 mm

Q. (2) Calculate the rate of flow of water through a pipe of diameter 300 mm. when the loss of head due to friction between the two ends of pipe 400 m apart is 5 m. of water. Take the value of $f = 0.009$

Soln: Given: $d = 300 \text{ mm} = 0.3 \text{ m}$

$$L = 400 \text{ m} \quad , \quad f = 0.009$$

Loss of head due to friction $= h_f = 5$ m

As
$$h_f = \frac{4fLV^2}{2gd}$$

$$5 = \frac{4 \times 0.009 \times 400 \times V^2}{2 \times 9.81 \times 0.3}$$

$$V^2 = 2.0438$$
$$V = 1.43 \text{ m/s}$$

Rate of flow
$$Q = A \times V$$
$$= \frac{\pi}{4} d^2 \times V$$
$$= \frac{\pi}{4} \times (0.3)^2 \times 1.43$$
$$= 0.101 \text{ m}^3/\text{s}$$

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Q. (3) Derive an expression for the time of emptying a tank of uniform cross-section through an orifice at its bottom.

Soln:

Let, A = Area of tank

a = Area of orifice

h_1 = Initial height of the liquid

h_2 = Final height of the liquid

T = Time in seconds for the liquid to fall from height h_1 to h_2

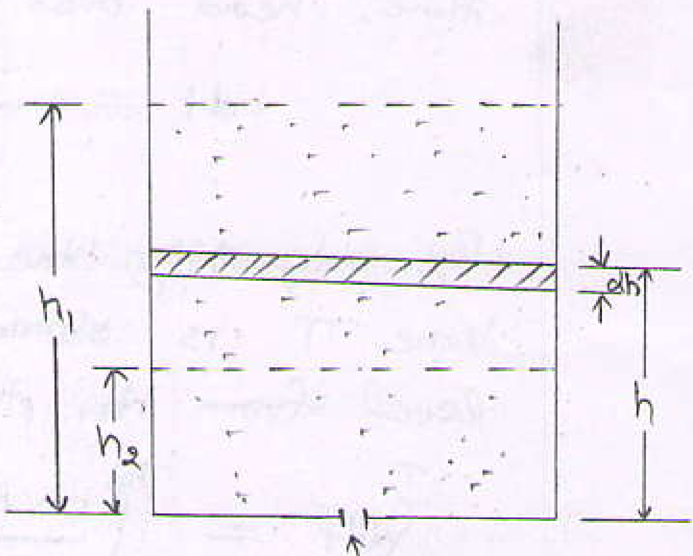

Fig: Tank of uniform cross-section with an orifice at its bottom

Let at any time, h be the height of the liquid above the orifice. Let dh be the decrease in the liquid surface in a small interval of time dt.

Volume of liquid leaving the tank in time dt,

$$dQ = A \times dh$$

Discharge through orifice in time dt

$$dQ = C_d \times \text{area} \times \text{Velocity} \times dt$$

$$= C_d \times a \times \sqrt{2gh} \times dt$$

Volume of liquid leaving the tank is equal to discharge through the orifice

$$\therefore \quad C_d \times a \times \sqrt{2gh} \times dt = -A \times dh$$

Here -ve sign indicates that with increase in time, head over the orifice decreases.

$$dt = \frac{-A \times dh}{C_d \times a \times \sqrt{2gh}}$$

By integrating the above equation, the total time T is obtained to lower the liquid level from h_1 to h_2.

$$\int_0^T dt = \int_{h_1}^{h_2} \frac{-A\,dh}{C_d \times a \times \sqrt{2gh}} = -\frac{A}{C_d \times a \times \sqrt{2g}} \int_{h_1}^{h_2} \frac{dh}{\sqrt{h}}$$

$$\left[t\right]_0^T = -\frac{A}{C_d \times a \times \sqrt{2g}} \left[\frac{\sqrt{h}}{1/2}\right]_{h_1}^{h_2}$$

$$T = -\frac{2A}{C_d \times a \times \sqrt{2g}} \left[\sqrt{h_2} - \sqrt{h_1}\right]$$

$$T = \frac{2A\left(\sqrt{h_1} - \sqrt{h_2}\right)}{C_d \times a \times \sqrt{2g}}$$

If tank is completely empty, then $h_2 = 0$

$$\therefore \quad T = \frac{2A\sqrt{h_1}}{C_d \times a \times \sqrt{2g}}$$