II Test EE-205 (ECT)

Attempt any three questions-

Q. 1. Explain the following-
(i) Resonance frequency
(ii) Bandwidth
(iii) Q-factor
(iv) selectivity

Q. 2. In a series R-L-C circuit, $R = 10\,\Omega$, $L = 0.1\,H$, $C = 8\,\mu F$
calculate - (i) Resonance frequency (ii) Q-factor
(iii) Half power frequencies (iv) Bandwidth

Q. 3. Find Laplace Transform of $\cos(\omega t + \theta)$

Q. 4. Find Inverse Laplace Transform of $F(s) = \frac{1}{s(s^2+4s+4)}$

Ans. 1. (i) In a series R-L-C ckt.
at resonance $X_L = X_C$, $\omega_0 L = \frac{1}{\omega_0 C}$

$$\omega_0^2 = \frac{1}{LC}, \quad \omega_0 = \frac{1}{\sqrt{LC}}, \quad 2\pi f_0 = \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

f_0 is resonance frequency at which resonance occurs.

(ii) Frequencies at which current is $\frac{1}{\sqrt{2}}$ times of the max^m current at resonance are called half power frequencies f_1 & f_2

$$f_2 - f_1 = B.W. = \frac{R}{2\pi L}$$

(iii) Q-factor is defined as voltage magnification at resonance

$$\text{Q-factor} = \frac{\text{voltage drop across L or C at resonance}}{\text{Supply voltage at resonance}}$$

$$= \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega_0 L}{R}$$

$$= 2\pi f_0 \frac{L}{R} = 2\pi \cdot \frac{L}{R} \cdot \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

or $\text{Q-factor} = 2\pi \times \frac{\text{Maximum energy stored}}{\text{energy dissipated per cycle}}$

(iv) Selectivity - It indicates the ability of a circuit to select a particular bandwidth. It is the ratio of the bandwidth to the resonant frequency. Thus

$$\text{selectivity} = \frac{f_2 - f_1}{f_0} = \frac{B.W.}{f_0}$$

Ans. 2. $R = 10\ \Omega, \quad L = 0.1\ H, \quad C = 8\ \mu F$

(i) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 8 \times 10^{-6}}}$

$= 178\ Hz$

(ii) $Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{10}\sqrt{\frac{0.1}{8 \times 10^{-6}}} = 11.18$

(iii) $f_1 = f_0 - \frac{R}{4\pi L}$

$= 178 - \frac{10}{4\pi \times 0.1} = 170\ Hz.$

$f_2 = f_0 + \frac{R}{4\pi L}$

$= 178 + \frac{10}{4\pi \times 0.1} = 186\ Hz$

(iv) $B.W. = f_2 - f_1$

$= 186 - 170 = 16\ Hz$

Ans. 3. $f(t) = \cos(\omega t + \theta)$

$F(s) = L[\cos(\omega t + \theta)]$

$= L[\cos \omega t \cos\theta - \sin \omega t \sin\theta]$

$= L[\cos \omega t \cos\theta] - L[\sin \omega t \sin\theta]$

$= \cos\theta\, L[\cos \omega t] - \sin\theta\, L[\sin \omega t]$

$= \cos\theta \cdot \frac{s}{s^2 + \omega^2} - \sin\theta \cdot \frac{\omega}{s^2 + \omega^2}$

$= \frac{s\cos\theta - \omega\sin\theta}{s^2 + \omega^2}$

Ans. 4. $F(s) = \dfrac{1}{s(s^2+4s+4)}$

$$\frac{1}{s(s^2+4s+4)} = \frac{1}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$\frac{1}{s(s+2)^2} = \frac{A(s+2)^2 + Bs(s+2) + Cs}{s(s+2)^2}$$

$$1 = A(s+2)^2 + Bs(s+2) + Cs$$

$$1 = A(s^2+4s+4) + B(s^2+2s) + Cs$$

$$1 = s^2(A+B) + s(4A+2B+C) + 4A$$

comparing coefficients of s^2, s & constant

$$4A = 1 \quad , \quad A = \frac{1}{4}$$

$$A + B = 0 \qquad B = -A = -\frac{1}{4}$$

$$4A + 2B + C = 0$$

$$4\left(\frac{1}{4}\right) + 2\left(-\frac{1}{4}\right) + C = 0$$

$$1 - \frac{1}{2} + C = 0$$

$$\frac{1}{2} + C = 0 \qquad C = -\frac{1}{2}$$

Putting values of A, B & C

$$F(s) = \frac{1}{s(s+2)^2} = \frac{1}{4s} - \frac{1}{4}\cdot\frac{1}{s+2} - \frac{1}{2}\cdot\frac{1}{(s+2)^2}$$

Taking Inverse Laplace Transform

$$L^{-1}[F(s)] = L^{-1}\left[\frac{1}{4s} - \frac{1}{4(s+2)} - \frac{1}{2(s+2)^2}\right]$$

$$= L^{-1}\left[\frac{1}{4s}\right] - L^{-1}\left[\frac{1}{4(s+2)}\right] - L^{-1}\left[\frac{1}{2(s+2)^2}\right]$$

$$= \frac{1}{4}L^{-1}\left[\frac{1}{s}\right] - \frac{1}{4}L^{-1}\left[\frac{1}{s+2}\right] - \frac{1}{2}L^{-1}\left[\frac{1}{(s+2)^2}\right]$$

$$f(t) = \frac{1}{4} - \frac{1}{4}e^{-2t} - \frac{1}{2}te^{-2t}$$