Date: 20/1/18

CE-201 (GPC-Karauli)

2nd Test

Max^m Marks-15

(01)

Q1. Write a short Note on :- (05)

(i) Modulus of elasticity (E) - 02

(ii) Poisson's Ratio (μ) - 02

(iii) Bulk Modulus (K) - 01

Q2. Prove that

(05)

$$\boxed{E = 3K(1-2\mu)}$$

K = Bulk Modulus

E = Young's Modulus

μ = Poisson Ratio.

Q3 (05) The stress on two perpendicular planes through a point are 60 N/mm² (tension) & 40 N/mm² Compression and 30 N/mm² shear. Find the stress components and the resultant stress on a plane at 60° to that of tensile stress.

Q4 (05) A piece of material is subjected to tensile stress of 70 N/mm² and 50 N/mm² at right Angles to each other. Find fully the stress on a plane the normal of which makes an angle of 35° with 70 N/mm² stress.

Sd/-

Sol Ans-1

(1) Modulus of elasticity (E) -

According to Hook's Law.

$$\sigma = E \times \varepsilon_0$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0} = \frac{\text{Direct stress}}{\text{Diret strain}}}$$

Note(1) It is an elastic constant which is used to determine linear strains in material

(a) for all type of steel $E = 2\times10^5$ N/mm2.

(2) poisson's ratio (μ)

$$\mu = \left|\frac{-\text{lateral strain}}{\text{longitudnal strain}}\right|$$

Note: ① It is used to determine lateral strain in elastic material

(a) for material $\mu_{min} = 0$

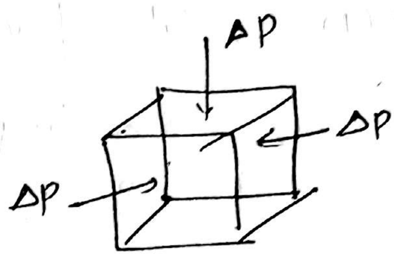
$\mu_{max} = .5$

(3) $$\mu = -\frac{\text{lateral strain}}{\left(\frac{\sigma}{E}\right)}$$

$$\text{lateral strain} = -\mu\left(\frac{\sigma}{E}\right)$$

(iii) Bulk Modulus (K) :-

It is used to determine volumetric strain in materials.



$$\boxed{K = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}}$$

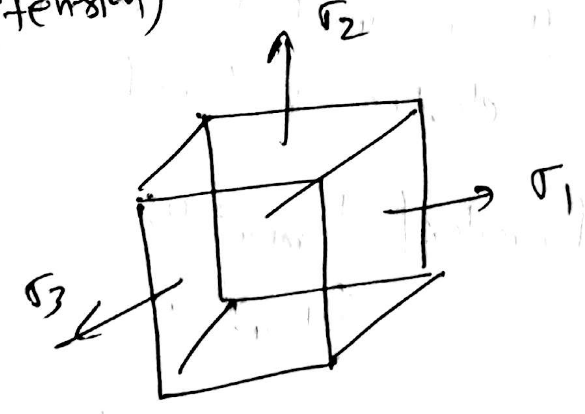
$$K = \frac{\text{change in stress}}{\text{volumetric strain}}$$

Note! (1) Compressibility of material $= \frac{1}{K}$

Ans-2 we have to prove

$$E = 2K(1-2\mu)$$

let us consider stress element with 3-mutually perpendicular stress (tension)



x-direction strain

$$\varepsilon_x = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E} \quad -(1)$$

y-direction strain

$$\varepsilon_y = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E} - \frac{\mu\sigma_3}{E} \quad -(2)$$

ces as in sprin... control motions ... springs

z- direction strain

$$\varepsilon_z = \frac{\sigma_3}{E} - \frac{\mu\sigma_1}{E} - \frac{\mu\sigma_2}{E} \quad \text{--- } 3$$

03

Vol^m strain.

$$e_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$e_v = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E} + \frac{\sigma_2}{E} - \frac{\mu\sigma_2}{E} - \frac{\mu\sigma_3}{E} + \frac{\sigma_3}{E} - \frac{\mu\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

$$= \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{E}\right) - \left(\frac{2\mu\sigma_1}{E} + \frac{2\mu\sigma_2}{E} + \frac{2\mu\sigma_3}{E}\right)$$

$$= \left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{E}\right) - \frac{2\mu}{E}(\sigma_1 + \sigma_2 + \sigma_3)$$

$$e_v = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{E}\left[1 - 2\mu\right] \quad \text{--- } 2$$

If $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ (In case of Hydrostatic pressure)

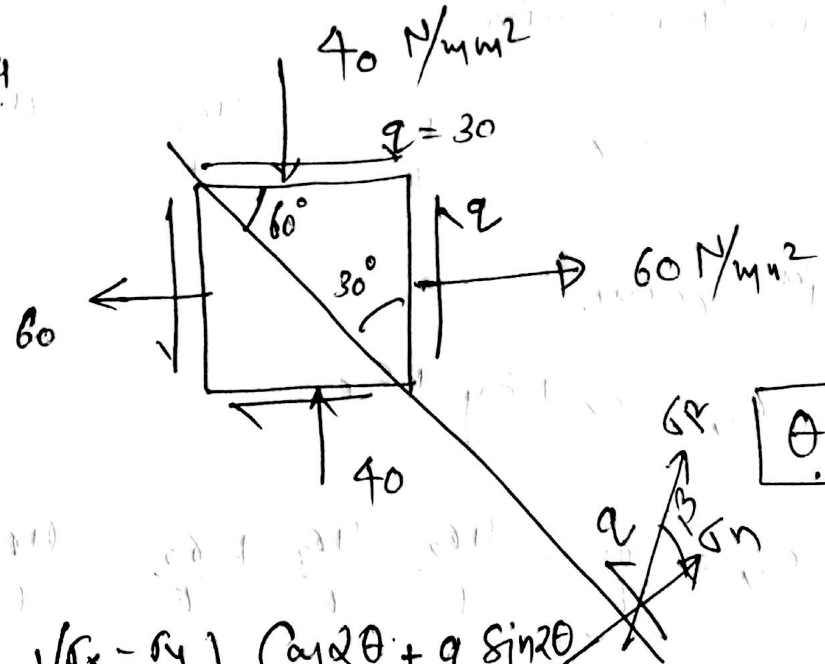
$$e_v = \frac{3\sigma}{E}(1 - 2\mu)$$

$$E = 3\frac{\sigma}{e_v}(1 - 2\mu)$$

$$\boxed{E = 3K(1 - 2\mu)}$$

Hence prove.

Q3 Given data



$\boxed{\theta = 30^\circ}$

We know that

Normal stress $\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta + q \sin 2\theta$

$= \left(\frac{60 - 40}{2}\right) + \left(\frac{60+40}{2}\right) \cos 60^\circ + 30 \sin 60$

$= \boxed{10.98 \text{ N/mm}^2}$

Shear stress

$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + q \cos 2\theta$

$= -\left(\frac{60+40}{2}\right) \sin 60 + 30 \cos 60 = \boxed{-58.3 \text{ N/mm}^2}$

Resultant stress

$\sigma_R = \sqrt{\sigma_n^2 + \tau^2 + \cancel{2\sigma_n \tau \times \cos 90}}$ 0

$= \sqrt{(10.98)^2 + (58.3)^2} = 59.33 \text{ N/mm}^2$

direction

$\tan \beta = \frac{q}{\sigma_n} = \frac{-58.3}{10.98}$ | $\beta = \cancel{10.67^\circ}$ 79.33° Ans

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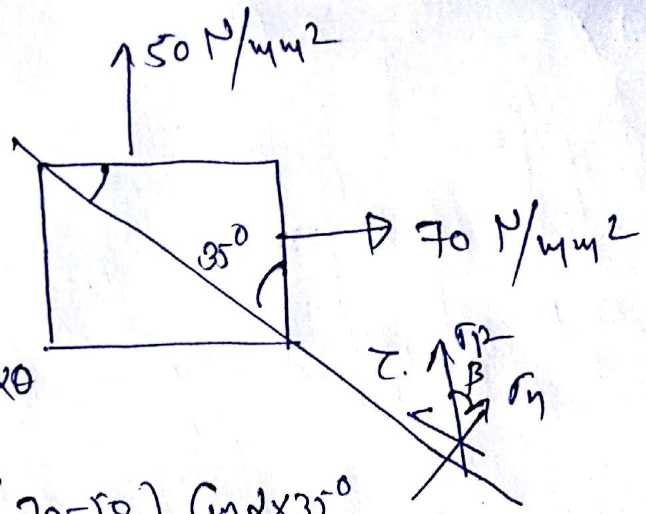
Ans-4 Given data.



Normal stress

$$\sigma_n = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta$$

$$= \left(\frac{70+50}{2}\right) + \left(\frac{70-50}{2}\right)\cos 2\times 35^0$$

$$= 63.42 \text{ N/mm}^2$$

shear stress

$$\tau = -\frac{(\sigma_1-\sigma_2)}{2}\sin 2\theta = \left(\frac{70-50}{2}\right)\sin 70^\circ = 9.4 \text{ N/mm}^2$$

Resultant

$$\sigma_R = \sqrt{\sigma_n^2+\tau^2} = \sqrt{63.42^2+9.4^2} = 64.11 \text{ N/mm}^2$$

Direction

$$\tan\beta = \frac{9.4}{63.4}$$

$$\beta = \tan^{-1}\left(\frac{9.4}{63.4}\right)$$