

Q: ① निम्न का मान ज्ञात करें; (Find the value of following):

(a) $\begin{bmatrix} 3x+4y & x-2y \\ a+b & 2a-b \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & -5 \end{bmatrix}$ { a तथा b का मान = ? }

Solⁿ:
 $a+b = 5$ — ①

$2a-b = -5$ — ②

eq ① + eq ② $\Rightarrow 3a = 0 \Rightarrow \boxed{a=0}$

Put the value of a in eq ① $\Rightarrow 0+b=5 \Rightarrow \boxed{b=5}$

(b) $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ then $A^2 = ?$

Solⁿ:
 $A^2 = A \times A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$
 $= \begin{bmatrix} (1)(1) + (-2)(2) + (3)(-3) & (1)(-2) + (-2)(3) + (3)(1) & (1)(3) + (-2)(-1) + (3)(2) \\ (2)(1) + (3)(2) + (-1)(-3) & (2)(-2) + (3)(3) + (-1)(1) & (2)(3) + (3)(-1) + (-1)(2) \\ (-3)(1) + (1)(2) + (2)(-3) & (-3)(-2) + (1)(3) + (2)(1) & (-3)(3) + (1)(1) + (2)(2) \end{bmatrix}$
 $= \begin{bmatrix} 1-4-9 & -2-6+3 & 3+2+6 \\ 2+6+3 & -4+9-1 & 6+(-3)-2 \\ -3+2-6 & -6+3+2 & -9+1+4 \end{bmatrix} = \begin{bmatrix} -12 & -5 & 11 \\ 11 & 4 & 1 \\ -7 & 11 & -6 \end{bmatrix}$ Ans

Q: ② निम्न को सिद्ध करें; - (Prove the following):

(a) If $A = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ तथा $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ तो $(AB)^{-1} = B^{-1}A^{-1}$

Solⁿ:
 $AB = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} (2)(1) + (0)(0) & (2)(-1) + (0)(2) \\ (1)(1) + (2)(0) & (1)(-1) + (2)(2) \end{bmatrix}$
 $= \begin{bmatrix} 2+0 & -2+0 \\ 1+0 & -1+4 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}$

$$|AB| = \begin{vmatrix} 2 & -2 \\ 1 & 3 \end{vmatrix} = (2)(3) - (-1)(-2) = \cancel{6-2} 6+2=8$$

$$(AB)^{-1} = \frac{\text{Adj}(AB)}{|AB|} = \frac{\begin{bmatrix} 3 & -2 \\ +1 & 2 \end{bmatrix}^T}{(8)} = \frac{1}{8} \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix} \text{--- (1)}$$

$$= \frac{1}{8} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix} \text{--- (1)}$$

$$\text{Adj}(A) = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

$$|A| = (2)(2) - (1)(0) = 4 - 0 = 4$$

$$(A)^{-1} = \frac{1}{4} \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$$

$$\text{Adj}(B) = \begin{bmatrix} 2 & 0 \\ +1 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & +1 \\ 0 & 1 \end{bmatrix}$$

$$|B| = (1)(2) - (0)(-1) = 2 - 0 = 2$$

$$(B)^{-1} = \frac{1}{2} \begin{bmatrix} 2 & +1 \\ 0 & 1 \end{bmatrix}$$

$$(B)^{-1}(A)^{-1} = \frac{1}{2} \begin{bmatrix} 2 & +1 \\ 0 & 1 \end{bmatrix} \cdot \frac{1}{4} \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} (2)(2) + (1)(-1) & (2)(0) + (1)(2) \\ (0)(2) + (1)(-1) & (0)(0) + (1)(2) \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} 4-1 & 0+2 \\ 0-1 & 0+2 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix} \text{--- (2)}$$

$$= \frac{1}{8} \begin{bmatrix} 4-1 & 0+2 \\ 0-1 & 0+2 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix} \text{--- (2)}$$

by eqⁿ & (2) LHS = R.H.S. N.P.

$$(b) \begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix} = -(a+b+c)(a-b)(b-c)(c-a)$$

$$\text{LHS} \begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix} \quad C_1 \rightarrow C_1 + C_2 \Rightarrow \begin{vmatrix} a+b+c & b+c & a^2 \\ a+b+c & c+a & b^2 \\ a+b+c & a+b & c^2 \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & b+c & a^2 \\ 1 & c+a & b^2 \\ 1 & a+b & c^2 \end{vmatrix} \quad \begin{array}{l} R_1 \rightarrow R_1 - R_2 \\ R_2 \rightarrow R_2 - R_3 \end{array}$$

$$= (a+b+c) \begin{vmatrix} 0 & b-a & a^2-b^2 \\ 0 & c-b & b^2-c^2 \\ 1 & a+b & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c) \begin{vmatrix} 0 & -1 & a+b \\ 0 & -1 & b+c \\ 1 & a+b & c^2 \end{vmatrix} \quad R_2 \rightarrow R_2 - R_1$$

$$= (a+b+c)(a-b)(b-c) \begin{vmatrix} 0 & -1 & a+b \\ 0 & 0 & c-a \\ 1 & a+b & c^2 \end{vmatrix}$$

$$= (a+b+c)(a-b)(b-c)(c-a) \begin{vmatrix} 0 & -1 & a+b \\ 0 & 0 & c-a \\ 1 & a+b & c^2 \end{vmatrix} \quad R_2 \text{ से उत्तर निकालें।}$$

$$= (a+b+c)(a-b)(b-c)(c-a) [-1 [0 - (-1)]] a$$

$$= -(a+b+c)(a-b)(b-c)(c-a) = \frac{\text{R.H.S.}}{\text{H.P.}}$$

Q: 5 निम्न समीकरणों का हल करें:-

Solve the following system of equations:

$$x+y+z = 6$$

$$x+2y+3z = 14$$

$$-x+y-z = -2$$

Sol:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & -1 \end{vmatrix} = 1[-2-3] - 1[-1+3] + 1[1+2] \\ = -5 - 2 + 3 = -4 \Rightarrow \boxed{\Delta = -4}$$

$$\Delta x = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & 14 \\ 1 & -1 & -2 \end{vmatrix} = 1[-6+14] - 1[-4-14] + 6[-2-3]$$

$$= 8 + 18 - 30 = -4 \Rightarrow \boxed{\Delta x = -4}$$

$$\Delta y = - \begin{vmatrix} 1 & 1 & 6 \\ 1 & 3 & 14 \\ -1 & -1 & -2 \end{vmatrix} = - [1(-6+14) - 1(-2+14) + 6(-1+3)]$$

$$= - [8 - 12 + 12] = -8 \Rightarrow \boxed{\Delta y = -8}$$

$$\Delta z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 14 \\ -1 & 1 & -2 \end{vmatrix} = 1[-4-14] - 1[-2+14] + 6[1+2]$$

$$= -18 - 12 + 18 = -12 \Rightarrow \boxed{\Delta z = -12}$$

$$x = \frac{\Delta x}{\Delta} = \frac{-4}{-4} = 1 \Rightarrow \boxed{x = 1}$$

$$y = \frac{\Delta y}{\Delta} = \frac{-8}{-4} = 2 \Rightarrow \boxed{y = 2}$$

$$z = \frac{\Delta z}{\Delta} = \frac{-12}{-4} = 3 \Rightarrow \boxed{z = 3}$$

$$[a_1]x + [a_2]y + [a_3]z = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{vmatrix} = \Delta$$