Subject Code - 205

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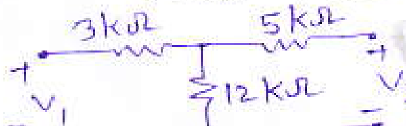
Q.1 Find the laplace transform of $e^{-at}\sin bt$?

Q.2 Draw the Pole-Zero diagram - $F(s) = \frac{S(S+1)}{S^2+2S+2}$

Q.3 Convert Z-Parameter into H-Parameter & find the Z-Parameter of the circuit?



1.) Find the laplace transform of $e^{-at}\sin bt$.

$\Rightarrow$ $f(t) = e^{-at}\sin bt$

we know that $L[f\,t] = \int_0^\infty f(t)\, e^{-st}\, dt$

$\therefore$ $\overset{a \to a}{L\,f(t)} = \int_0^\infty e^{-at}\, e^{-st} \sin bt \, dt$ ~~let~~

Let, $I = \int_0^\infty \underset{\text{II}}{e^{-(s+a)t}}\, \underset{\text{I}}{\sin bt}\, dt$ --- (1)

$\because$ we know $\int \text{I}\,\text{II}\, dt = \text{I}\int \text{II}\, dt - \int\left\{\frac{d}{dt}\text{I}\int \text{II}\, dt\right\} dt$

$\therefore I = \sin bt \int_0^\infty e^{-(s+a)t}\, dt - \int_0^\infty \left\{\frac{d}{dt}\sin bt \int e^{-(s+a)t}\, dt\right\} dt$

$= \left[\frac{\sin bt \cdot e^{-(s+a)t}}{-(s+a)}\right]_0^\infty + \frac{b}{s+a}\int_0^\infty \underset{\text{I}}{\cos bt} \cdot \underset{\text{II}}{e^{-(s+a)t}}\, dt$

$= 0 + \frac{b}{s+a}\left[\cos bt \int_0^\infty e^{-(s+a)t}\, dt - \int_0^\infty \left\{\frac{d}{dt}\cos bt \int e^{-(s+a)t}\, dt\right\}\right]$

$= \frac{b}{s+a}\left[\left[\frac{\cos bt \cdot e^{-(s+a)t}}{-(s+a)}\right]_0^\infty - \frac{b}{s+a}\int_0^\infty \sin bt\, e^{-(s+a)t}\, dt\right\}\right]$

$I = \frac{b}{s+a}\left[\left(\frac{-1}{-(s+a)}\right) - \frac{b}{s+a} I\right]$

$I\left[1 + \frac{b^2}{(s+a)^2}\right] = \frac{b}{(s+a)^2}$

$$\boxed{I = \frac{b}{(s+a)^2 + b^2}}$$

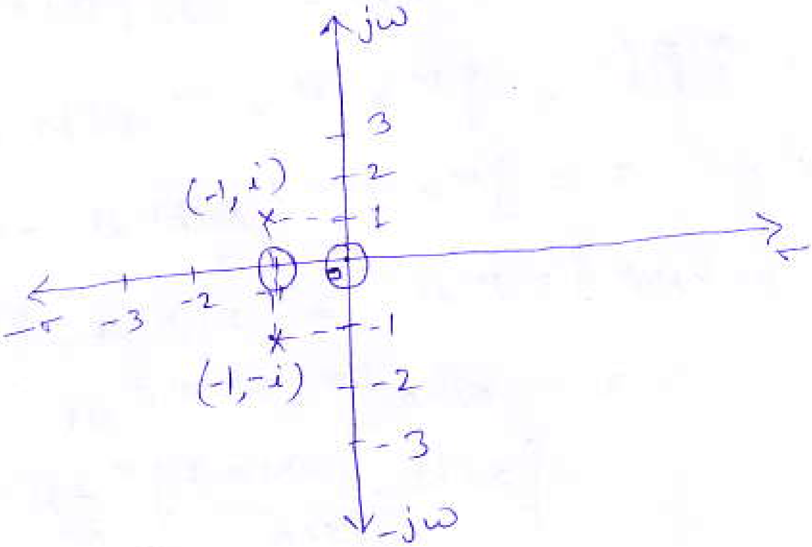
Ans. 2 $F(s) = \frac{s(s+1)}{s^2+2s+2}$

zero → $s = 0, -1$

Pole → $s = \frac{-2 \pm \sqrt{4-8}}{2}$

$s = \frac{-2 \pm \sqrt{-4}}{2}$

$s = \frac{-2 \pm 2i}{2}$ ⇒ $s = -1+i, -1-i$



Ans. 3 Z-Parameter:-

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — (1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — (2)

H-Parameter:-

$V_1 = H_{11} I_1 + H_{12} V_2$ — (3)

$I_2 = H_{21} I_1 + H_{22} V_2$ — (4)

from eq. (2)

$I_2 = -\left(\frac{Z_{21}}{Z_{22}}\right) I_1 + \frac{V_2}{Z_{22}}$ — (5)

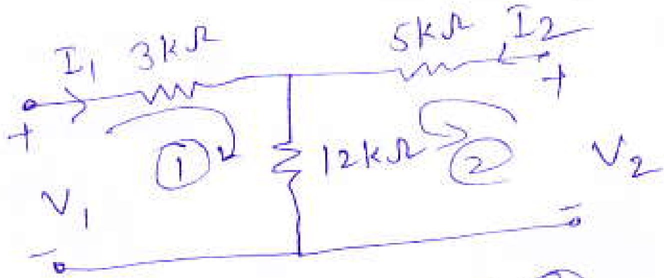
Put the value of 'I_2' from eq. (5) in eq. (1).

$$V_1 = Z_{11} I_1 + Z_{12}\left[-\left(\frac{Z_{21}}{Z_{22}}\right) I_1 + \frac{V_2}{Z_{22}}\right]$$

$$V_1 = I_1\left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{22}}\right] + \frac{Z_{12}}{Z_{22}} V_2$$

$$I_2 = -\left(\frac{Z_{21}}{Z_{22}}\right) I_1 + \frac{V_2}{Z_{22}}$$

$\Rightarrow$



Apply KVL at loop (1).

$$V_1 = 3\times10^3 I_1 + 12\times10^3 (I_1 + I_2)$$

$$V_1 = 15\times10^3 I_1 + 12\times10^3 I_2 \quad -(1)$$

Apply KVL at loop (2)

$$V_2 = 5\times10^3 I_2 + 12\times10^3 (I_1 + I_2)$$

$$V_2 = 12\times10^3 I_1 + 17\times10^3 I_2 \quad -(2)$$

from eq. (1) & (2), Z-Parameter will be -

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 15\times10^3 & 12\times10^3 \\ 12\times10^3 & 17\times10^3 \end{bmatrix} \Omega \quad \text{Ans.}$$