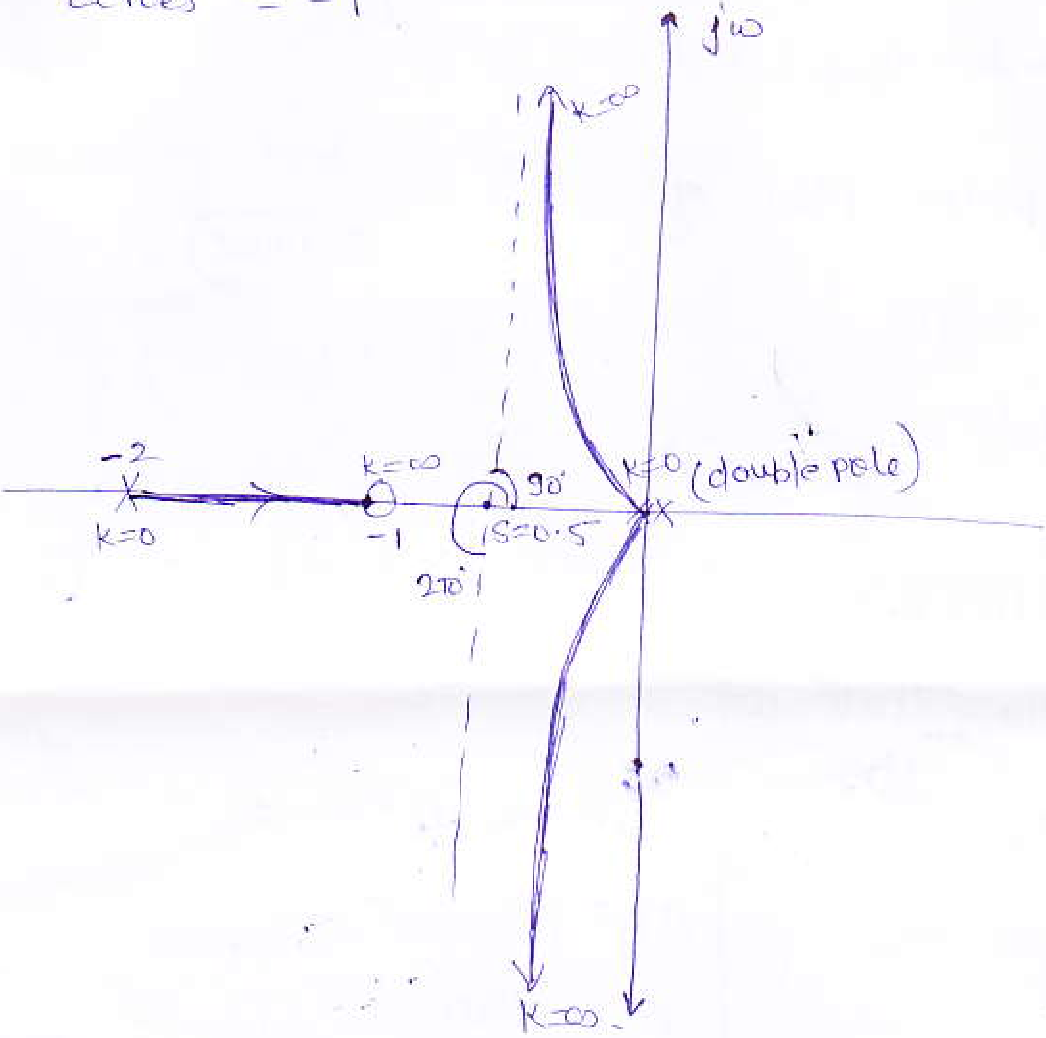
EE-305 [Control System]

III rd Mid term

(1) Sketch the root locus of $G(s) = \frac{K(s+1)}{s^2(s+2)}$

Poles $P = 0, 0, -2$

Zeroes $= -1$



No of asymptote = No of pole - No of zeroes

$= 3 - 1 = 2$

Asymptote angle $\alpha_0 = \frac{180(2K+1)}{P-Z} = \frac{180(2\times0+1)}{2} = \pm 90°$

$\alpha_1 = \frac{180(2\times1+1)}{2} = \pm 270°$

Intersection of asymptote $= \frac{\Sigma P - \Sigma Z}{P-Z} = \frac{-2-(-1)}{3-1} = -0.5$

The No. of root locus are three. The three root loci will originate from 3 poles. Out of 3, one root locus will terminate at the zero $s=-1$ and other 2 root loci will terminate at infinity as shown in figure by dark line.

Q.2 Plot the polar plot of . $G(s) = \frac{1}{s(1+sT)}$

Step 1 Putting $s = j\omega$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)}$$

Step 2 $M = \frac{1}{\omega\sqrt{1+\omega^2T^2}}$

$$\phi = -90^\circ - \tan^{-1}\omega T$$

Step 3 $\omega = 0$ पर

$M = \infty$

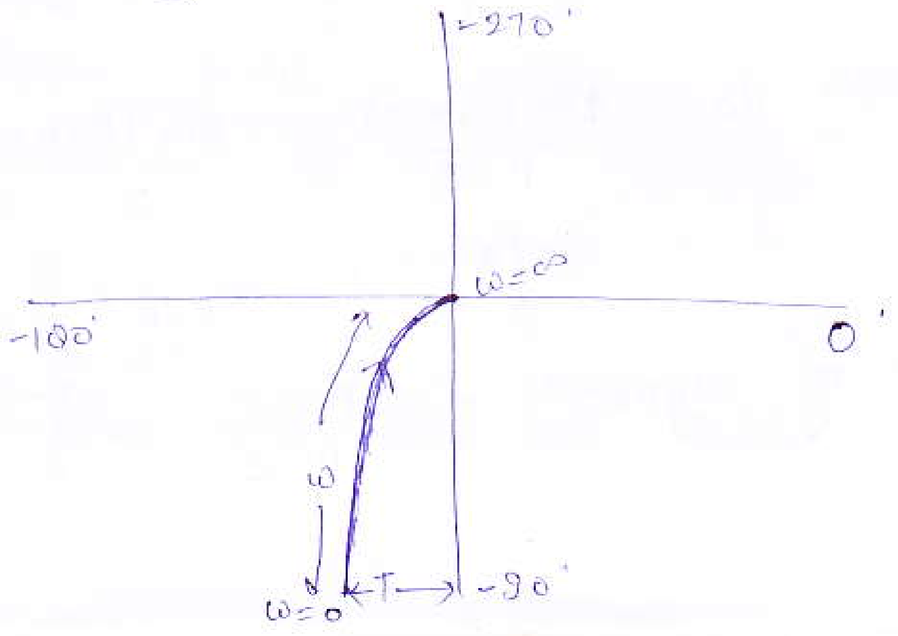
$\phi = -90^\circ$

$\omega = \infty$ पर

$M = 0^\circ$

$\phi = -180^\circ$

Step 4 Sketching the polar plot



ol Seperating $G(j\omega)$ into real and imaginary parts –

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)} \times \frac{-j\omega}{-j\omega} \times \frac{1-j\omega T}{1-j\omega T}$$

$$\Rightarrow G(j\omega) = \frac{-j\omega + j^2\omega^2 T}{-j^2\omega^2(1-j^2\omega^2 T^2)}$$

equating real part to zero.

$$\frac{-\omega^2 T}{-j^2\omega^2(1-j^2\omega^2 T^2)} = 0$$

$$\omega^2 = 0$$

$$\boxed{\omega = 0}$$

Hence we can get the value of M

$$M = \frac{1}{\omega\sqrt{1+\omega^2 T^2}} = \frac{1}{0\sqrt{1+0}} = \infty$$

Q.3. Sketch the nyquist plot of $G(s) = \frac{1}{s(1+s)(2+s)}$

Step 1 $$G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

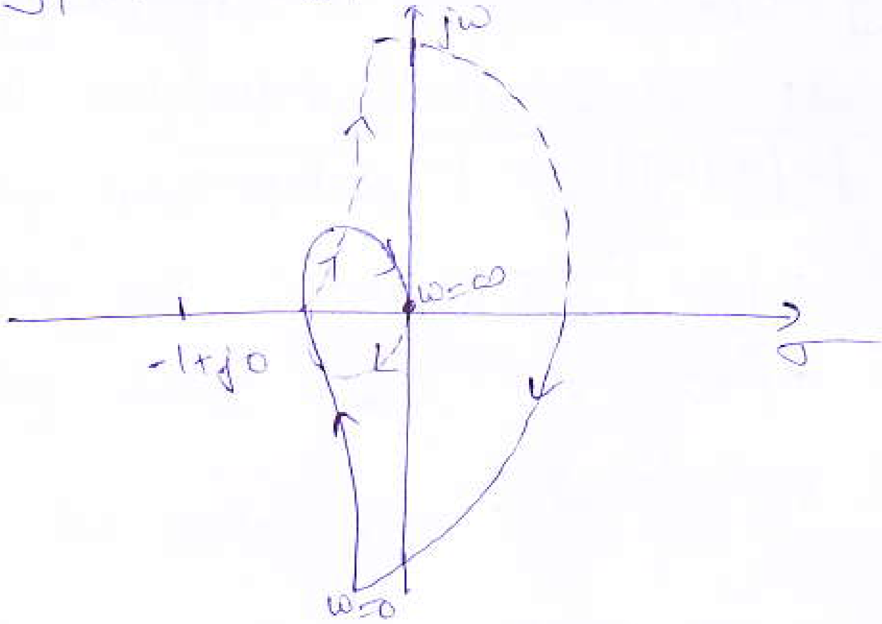
Step 2 $$M = \frac{1}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}}$$

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1} 2\omega$$

Step 3

$\omega = 0$ पर	$\omega = \infty$ पर
$M = \infty$	$M = 0$
$\phi = -90^\circ$	$\phi = -270^\circ$

Step 4 Nyquist is shown as -



Step 5 $$G(j\omega) = \frac{1 \times (-j\omega)(1-2j\omega)(1-j\omega)}{j\omega(1+2j\omega)(1+j\omega)}$$

$$= \frac{-3\omega - j(1-2\omega^2)}{j\omega(1+2j\omega)(1+j\omega)}$$

equating imaginary part to zero

$$1-2\omega^2 = 0$$

$$\omega^2 = \frac{1}{2} \Rightarrow \boxed{\omega = \frac{1}{\sqrt{2}} = 0.707}$$

hence $$M = \frac{1}{0.707\sqrt{1+\frac{1}{2}}\sqrt{1+2}} = 0.66$$

Step 6 $-1+j0$ is not encircled

hence $N=0$

$P=0$

we know $N = P - Z$

$0 = 0 - Z$

$Z = 0$

meance open and close loop stable.